

On the optimization of a CAPM-portfolio considering the possibility of safeguarding its loss

U. Spreitzer and V. Reznik

Dr. Dr. Heissmann GmbH

Abraham-Lincoln-Str. 22, 65191 Wiesbaden, Germany

Fon. +49-611-794-342

Fax. +49-611-794-209

Uli.spreitzer@heissmann.de

ABSTRACT

Using a portfolio built from bonds (investment without volatility) and shares (investment with volatility) corresponding to the CAPM we calculate the possible loss of this portfolio. The loss is measured by a so called lower partial moment of the rate of return of the portfolio. Using this loss, we optimize the composition of the portfolio with respect to this loss. Also we investigate the optimization of the portfolio, when the loss can be underwritten by an insurance. Concerning the premium of this insurance contract, we show, that when the premium is defined inadequate e.g. proportional to the investment or proportional to the amount of investment in shares, the optimal portfolio consists only of investment in shares. When the premium is defined more suitable e.g. proportional to the loss, the optimal portfolio is built by an investment in bonds and shares.

Keywords: Portfolio, CAPM, risk, insurance

1 INTRODUCTION

In a recent publications [1-3] we investigated the risk of an investment. We defined the risk as a return of the investment, below a defined value, which is demanded a priori. To measure the loss we defined a average loss L_A , using the so called lower partial moments, published by Fishburn [4]:

$$L_A = I \int_{-\infty}^{R_{nec}} (R_{nec} - R) p(R) dR$$

R_{nec} is the rate of return, which is necessary for the investor, I the amount of investment, R the rate of return of the investment and $p(R)$ is the probability density.

Concerning the investment, we investigated a portfolio consisting of bonds and shares. In this context share is an asset with a variable rate of return and bond an asset with an fixed rate of return. This investment is analogue to the investment corresponding to the capital market, as decribed in the Capital Asset Pricing Model (CAPM)[5]. Our model was a one periodic model.

With this method we obtain for the loss of such an portfolio L_A

$$L_A = I \int_{-\infty}^{R_{nec}} (R_{nec} - \alpha R_0 - (1 - \alpha) R_{TP}) p(\alpha R_0 + (1 - \alpha) R_{TP}) d(\alpha R_0 + (1 - \alpha) R_{TP})$$

we define:

$$p_{PF}(R_{TP}) \equiv p(\alpha R_0 + (1 - \alpha) R_{TP})$$

and we will use only $p_{PF}(R_{TP})$ and we call this $p(R_{TP})$. From this we conclude:

$$L_A = I(1 - \alpha) \int_{+\infty}^{\frac{R_{nec} - \alpha R_0}{1 - \alpha}} (R_{nec} - \alpha R_0 - (1 - \alpha) R_{TP}) p(R_{TP}) dR_{TP}$$

When we assume, that the shares are normally distributed, we obtain

$$L_A = I \frac{(1-\alpha)}{\sqrt{2\pi}\sigma_{TP}} \int_{-\infty}^{\frac{R_{nec}-\alpha R_0}{1-\alpha}} (R_{nec} - \alpha R_0 - (1-\alpha)R_{TP}) \exp\left[-\frac{(R_{TP} - \mu_{TP})^2}{2\sigma_{TP}^2}\right] dR_{TP}$$

2 REDUCTION OF THE LOSS OF AN PORTFOLIO BY VARIATION OF THE COMPOSITION

To obtain the composition of the portfolio with minimal L_A , one has to make variation of L_A with respect to α . Instead of doing this in an analytic manner, which is very difficult, we use numerical methods for two distinguished portfolios. We use values of the MSCI performance index for the years 1970 to 2002 to describe the characteristic values of the shares of the portfolios, as expected rate of return and variation.

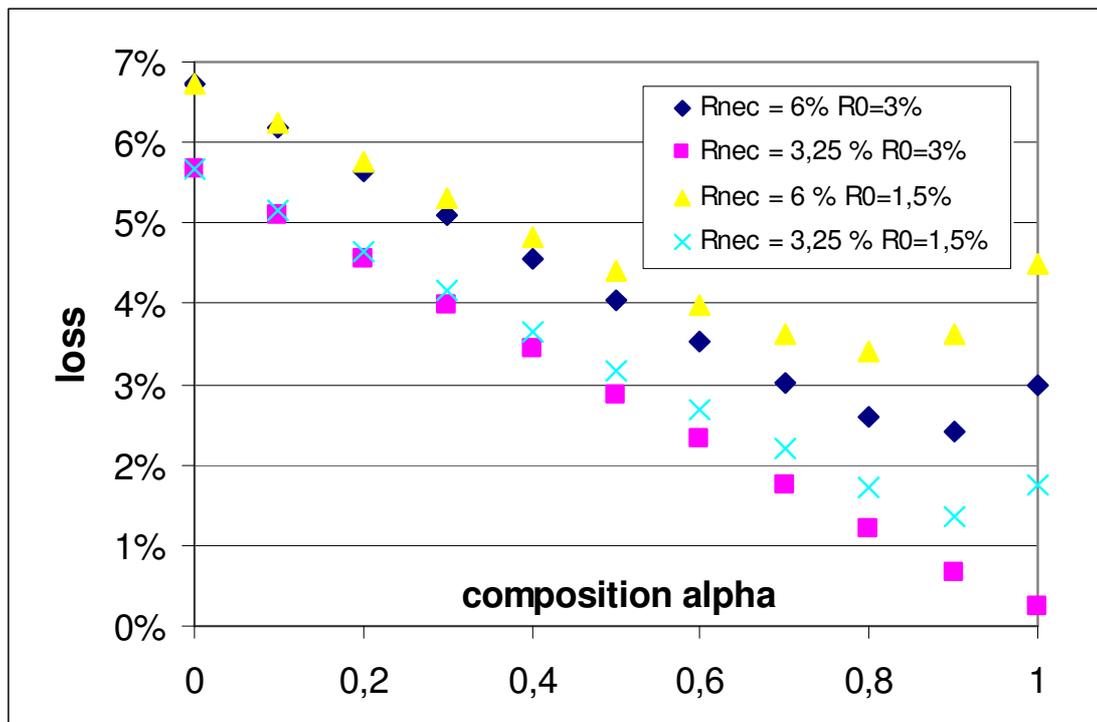


Fig. 1 L_A of a CAPM-portfolio (shares described by MSCI EUROPE Performance Index)

Of the different MSCI-Europe-ET-performance-indexes we use, that one, which is measured every month. From this we have evaluated by least mean square root deviation the value of the average rate of return and the variance. Assuming, that the rate of return is normal

distributed, we receive an average rate of return of 10.8 % and a variance of 20 %. Concerning the rate of return of the bonds we used 3 % and 1.5 % and a variance of 0%. Concerning R_{nec} we used values of 6 % (discount rate of book reserves of pensions in Germany) and 3,25 % (discount rate of insurance contracts up to 2002 in Germany).

As seen in figure 1 the average loss L_A has not only a minimum at $\alpha = 1$, i.e. for investment in bonds only, but also an minimum at 0,8 and 0,9.

This means that by investing not only in bonds but with some ratio of the investment in shares the average loss can be reduced whereby the expected rate of return can be increased compared to an investment in bonds only.

3 SAFEGUARDING OF THE LOSS AND OPTIMIZATION OF AN CAPM-PORTFOLIO

We now investigate the scenario, in which we can safeguard the average loss. We assume a insurance contract, from which we obtain an amount of $I (R_{nec} - R_{PF})$. This means, when the rate of return of the portfolio is below the rate R_{nec} the investor obtains this amount. E.g. in GB such a minimal rate of return of the contributions of the employees is guaranteed.

Concerning the premium of this insurance contract, we investigate three scenarios:

Scenario A: the premium P is proportional (factor π) to the investment I :

$$P = \pi I.$$

Such a premium seems to be inadequate, but it is used e.g. at the Pensionsversicherungsverein auf Gegenseitigkeit PSVaG. This institutions is obligative in Germany for safeguarding retirement benefits, for which the assets are within the companies (book reserves) or in pensions funds. Using a pay as you go finance, the PSVaG demands a premium proportional to the value of the obligation (measured by actuarial methods)

Scenario B: the premium P is proportional to the investment in shares:

$$P = \pi I (1-\alpha)$$

Scenario C: the premium P is proportional to the loss:

$$P = \pi L_A$$

With these scenarios we define an utility function and we assume, that the investor builds his portfolio in correspondance to the maximum of the utility function.

We define the utility function U as sum of the expected rate of return of the investment minus the premium of the insurance contract:

$$U = I (\alpha R_0 + (1-\alpha)R_{TP}) - P$$

Concerning the scenario A we obtain that an investor will always choose a portfolio which contains only shares and no bonds. This can be seen from:

$$U = I (\alpha R_0 + (1-\alpha)R_{TP}) - I \pi = I (\alpha R_0 + (1-\alpha)R_{TP} - \pi)$$

Assuming $R_{TP} > R_0$, one obtains the maximum of U for $\alpha = 0$. This means an investor will always choose a portfolio which contains only shares. Assuming $R_{TP} < R_0$, one obtains the maximum of U for $\alpha = 1$. This means an investor will always choose a portfolio which contains only bonds

Concerning the scenario B we investigate:

$$U = I (\alpha R_0 + (1-\alpha)R_{TP}) - (1-\alpha)I \pi = I (\alpha R_0 + (1-\alpha)(R_{TP} - \pi))$$

Assuming $R_{TP} - \pi > R_0$, one obtains the maximum of U for $\alpha = 0$. As seen above this means an investor will always choose a portfolio which contains only shares and assuming $R_{TP} - \pi < R_0$, one obtains the maximum of U for $\alpha = 1$ and an investor will always choose a portfolio which contains only bonds.

Concerning the scenario C we investigate:

$$U = I (\alpha R_0 + (1-\alpha)R_{TP}) - L_A$$

One can see easily from the calculation of L_A , that this optimization of L_A can at least not easily – done in an analytic manner. Because of that we made numerical calculation and optimization of U .

The portfolio is the portfolio as discussed before (MSCTI-performance etc.). Concerning the premium we used 3 premiums 0,5, 1 and 2 time LA.

As seen in fig. 2 for a premium, which is too low (0,5 times LA) or exactly "fair" (1 times LA) a prudent man will also only invest in shares. Using a higher premium the optimal portfolio with respect to U consists not only of shares but also of some bonds.

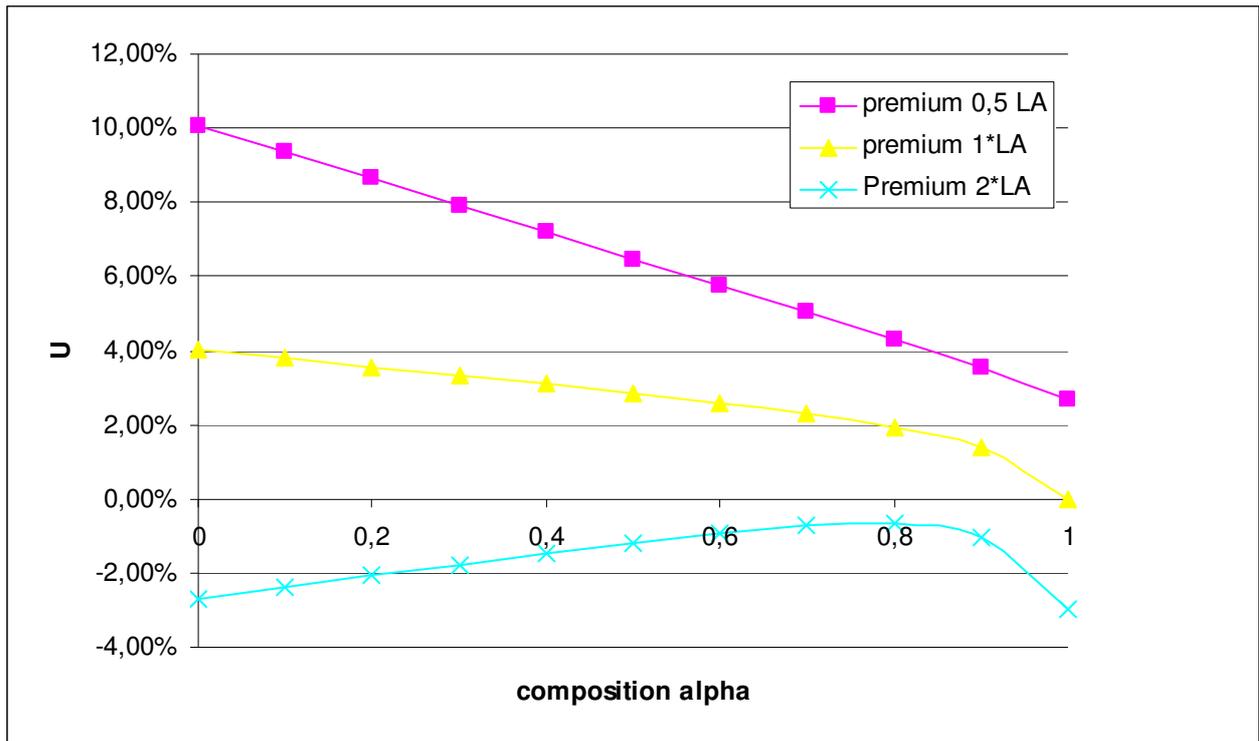


Fig. 2: Utility function of a CAPM portfolio, if the average loss L_A is safeguarded by different insurance premiums, R_{nec} 6%, $R_0 = 3\%$

4 CONCLUSION

We have shown, that the optimization of the rate of return of a portfolio can be done by using the measure of the average loss LA. Also we have shown how this portfolio can be optimized when, the loss of this portfolio is safeguarded by a insurance contract. And we have shown, that the definition of the premium for the insurance contract is very important for the composition of the portfolio. When the premium is not defined suitable as done with a premium proportional to the investment or proportional to the investment in shares, a prudent

man will always invest in shares only and the risk of this investment will be safeguarded by the insurance company. Because this insurance companies are obligative and executed by government institutions, the risk will be undertaken in consequence by all tax payers. Because of that, the premium, which is demanded by these institutions, should be more suitable. This can be done by a premium proportional to the loss of the investment.

ACKNOWLEDGEMENTS

The authors wish to thank Dr. Th. Riepl, Siemens VDO Regensburg for his support.

REFERENCES

- [1] Reznik V. Spreitzer U.W.; An investigation of a portfolio-loss under the CAPM; Economics Working Paper Archive at WUST series Finance; 12.4.2004; paper number 0402013.
- [2] Reznik, V. Spreitzer U.W.; An investigation of a portfolio-loss under CAPM; Proceedings of the 15th AFIR colloquium Zuerich CH; 6-9. 2005, Session B6;pensions.
- [3] Reznik V. Spreitzer U.W.; Investigation on risk of a CAPM-portfolio; Frühjahrstagung der Deutschen Physikalischen Gesellschaft, Berlin; 3.-9.3.2005; AKSOE 8.10.
- [4] Fishburn P., Mean-risk analysis with risk associated with below-target returns; American Economic Review; 1977; 67, 116-26.
- [5] Sharpe W.F., Capital Asset Prices: A Theory of Market Equilibrium under conditions of risk; The Journal of Finance; 1964; 19, 425-42.